C.3 Matrix Inverses

This is a guide to inverting 1×1 , 2×2 , and $n \times n$ matrices. Let *A* be the 1×1 matrix

$$A = \begin{bmatrix} a \end{bmatrix}$$
.

The inverse is simply the reciprocal:

$$A^{-1} = \begin{bmatrix} 1/a \end{bmatrix}.$$

Let *B* be the 2×2 matrix

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}.$$

It can be shown that the inverse follows a simple pattern:

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}$$
$$= \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix}.$$

Let *C* be an $n \times n$ matrix. It can be shown that its inverse is

$$C^{-1} = \frac{1}{\det C} \operatorname{adj} C,$$

where adj is the **adjoint** of *C*.

C.4 Euler's Formulas

Euler's formula is our bridge back-and forth between trigonomentric forms $(\cos \theta \text{ and } \sin \theta)$ and complex exponential form $(e^{j\theta})$:

$$e^{j\theta} = \cos\theta + j\sin\theta. \tag{C.15}$$

Here are a few useful identities implied by Euler's formula.

$$e^{-j\theta} = \cos\theta - j\sin\theta \tag{C.16a}$$

$$\cos\theta = \Re(e^{j\theta}) \tag{C.16b}$$

$$=\frac{1}{2}\left(e^{j\theta}+e^{-j\theta}\right) \tag{C.16c}$$

$$\sin \theta = \Im(e^{j\theta}) \tag{C.16d}$$

$$=\frac{1}{j2}\left(e^{j\theta}-e^{-j\theta}\right).$$
 (C.16e)



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C.5 Laplace Transforms

The definition of the one-side Laplace and inverse Laplace transforms follow.

Definition C.1: Laplace transforms (one-sided)

Laplace transform \mathcal{L} :

$$\mathcal{L}(y(t)) = Y(s) = \int_0^\infty y(t)e^{-st} dt.$$
(C.17)

Inverse Laplace transform \mathcal{L}^{-1} :

$$\mathcal{L}^{-1}(Y(s)) = y(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} Y(s) e^{st} ds.$$
(C.18)

See table B.1 for a list of properties and common transforms.

